

Interfering Doorway States and Giant Resonances. I. Resonance Spectrum and Multipole Strengths

V.V. Sokolov¹, I. Rotter^{2,3}, D.V. Savin¹ and M. Müller⁴

¹Budker Institute of Nuclear Physics, 6300090 Novosibirsk, Russia

²Forschungszentrum Rossendorf, Institut für Kern- und Hadronenphysik,
D-01314 Dresden, Germany

³Technische Universität Dresden, Institut für Theoretische Physik,
D-01062 Dresden, Germany

⁴Centro Internacional de Ciencias, Cuernavaca, Mexico

Abstract

A phenomenological schematic model of multipole giant resonances (GR) is considered which treats the external interaction via common decay channels on the same footing as the coherent part of the internal residual interaction. The damping due to the coupling to the sea of complicated states is neglected. As a result, the formation of GR is governed by the interplay and competition of two kinds of collectivity, the internal and the external one. The mixing of the doorway components of a GR due to the external interaction influences significantly their multipole strengths, widths and positions in energy. In particular, a narrow resonance state with an appreciable multipole strength is formed when the doorway components strongly overlap.

1 Introduction

In spite of much efforts, the nature of giant resonances (GR) is only partly understood nowadays. It is commonly accepted that they are collective excitations formed by coherent superpositions of many correlated predominantly one particle – one hole configurations with given total quantum numbers. These superpositions are usually found by diagonalizing the residual interaction in the $1p - 1h$ configuration space in the framework of the random phase (RPA) or Tamm-Dancoff approximations. But their detailed microscopic structure still needs further investigation.

The recent progress of high energy accelerators and high precision detectors gives the possibility to analyse in coincident experiments [1, 2, 3, 4, 5] the direct particle decays of the giant resonance states into specific channels. The decay rates extracted from the data contain the desired microscopic information. They are therefore a very useful tool for a careful study of the collective modes of nuclear motion.

At the GR energies, the value which is typical for the escape widths of the configurations contributing to the collective mode exceeds usually their level spacings. The energy continuum effects play therefore an important role and cannot be treated as a perturbation slightly broadening the levels which, nevertheless, remain isolated. A few methods exist to take into account the energy continuum in a straightforward manner. Some of them [6, 7, 8, 9, 10, 11] extend the RPA by including exactly the single particle continuum. The more general approach [12, 13, 14, 15] based on the explicit separation of the intrinsic and channel subspaces is closely related to Feshbach's idea of doorway states [16] which we exploit in the present paper.

Generally, a giant resonance consists of one or a few doorway states on the background of many complicated fine structure states. Even when within the RPA the corresponding doorway states are well isolated from one another, their overlapping with the background states can give rise to interference effects [17, 18] which cannot be described by a sum of independent Breit-Wigner contributions. However, due to the nearly chaotic structure of the background states, the main effect is [19, 20, 21], after energy averaging, the damping of the doorway states described by the spreading width. In contrast, the interference of the overlapping doorway states with each other may significantly influence the form of the energy spectrum of the decay products of giant resonances as shown in [19, 20].

In this paper we investigate the interference effects caused by the overlapping of doorway components of giant resonances. We use the extension, proposed in [19], of the simple Brown-Bolsterli [22] schematic model for multipole collective nuclear excitations to open (decaying) systems. Although being qualitative, this phenomenological model still maintains the main features of the real situation. The giant resonances emerge out of the interplay between two different kinds of collective behaviour: the synchronized collective intrinsic motion and the cooperative particle emission.

The present part I of our paper is devoted to the consideration of the collective excitation modes of the internal motion in an unstable nucleus while in the following part II [23] the cross section pattern is discussed.

Our schematic model along with its formal solution are presented in sect. 2. The two kinds of collectivity are analysed analytically in sect. 3 while their interplay and the interference of overlapping doorway resonances are described in sect. 4. Here, an important connection between the resonance spectrum and the multipole strengths of the doorway states is established.

The transition strengths as well as the escape widths get significantly redistributed between the overlapping doorway states when the interaction via the energy continuum becomes strong. The GR is mainly formed by two doorway states which acquire comparable parts of the total multipole strength. One of these states turns out to be much narrower than the other one. Both are lying at energies which are lower than the energy at which the giant resonance would lie if the mixing via the continuum would be absent.

In sect. 5, we show numerical results obtained in the same model with and without the restrictions introduced into the preceding analytical study. The results confirm the main features of the interference picture. At strong external coupling, they can be qualitatively understood in the two-level approximation introduced in sect. 4. The results are summarized in sect. 6.

We use the matrix shorthands throughout the paper. The capital letters are used for matrices in the Hilbert space of the internal motion; matrices in the space of the scattering channels are marked by the hat symbol. The *column* vectors in the internal space as well as the *row* vectors in the channel one are represented by bold letters.

2 The Model

As usual (see for example [24]), we suggest that a hierarchy of complexities of the internal states of the system under consideration exists. The first class contains the simplest states presumably of $1p - 1h$ nature which are directly connected to the continuum by appreciable transition matrix elements A_n^c . The states of the other classes of complexity have no direct connection to the continuum, at least to the same set of channel states as the first class. They can decay into these channels only through the states of the first class to which they are connected by some residual internal interaction.

In the present part of the paper, we restrict ourselves to the consideration of the dynamics of the states of the first class. The role of the background of complicated states will be discussed in the second part of our paper [23], referred in what follows as part II. As in the pioneering work by Brown and Bolsterli [22], we choose the Hamiltonian of the internal motion in the subspace of the states of the class 1 in the form

$$H = H_0 + \mathbf{D}\mathbf{D}^T. \quad (2.1)$$

It consists of the unperturbed part H_0 containing N discrete intrinsic levels $e_n (n = 1, 2, \dots, N)$, and of a factorized residual interaction of, let us say, dipole-dipole type. Besides by the internal residual interaction, the intrinsic levels are mixed also by the external interaction via common decay channels [25, 24, 26, 27]. This interaction,

$$W = AA^T \quad \Rightarrow \quad W_{mn} = \sum_{c(open)} A_m^c A_n^c, \quad (2.2)$$

originates from on-shell self-energy contributions of all open channels. The matrix A consists of k N -dimensional column vectors \mathbf{A}^c connecting all internal states with each channel c . These vectors are real because of time-reversal invariance. In the following we neglect a possible smooth energy dependence of the components A_n^c over the whole energy domain considered. The validity of such an assumption is not always obvious and deserves a special consideration.

Adding to H the external interaction (2.2) via common decay channels, we get the nonhermitian effective Hamiltonian

$$\mathcal{H} = H_0 + \mathcal{H}^{(int)} = H_0 + \mathbf{D}\mathbf{D}^T - \frac{i}{2} AA^T \equiv H - \frac{i}{2} W \equiv \mathcal{H}_0 + \mathbf{D}\mathbf{D}^T \quad (2.3)$$

which describes giant resonances as appearing out of the interplay of two kinds of collectivity, the internal and the external one. The expression (2.3) is the many-channel version of the effective Hamiltonian considered in reference [19]. Having omitted the coupling to the sea of complicated states, we neglect in particular the internal damping and spreading widths of the collective excitations formed by superpositions of the states of the first class which are embedded into the continuum. The coupling to the states of other classes will be considered in part II of our paper [23] by using statistical methods.

One finds due to the factorized form of the internal residual interaction

$$\text{Det}(\mathcal{E} - \mathcal{H}) = \text{Det}(\mathcal{E} - \mathcal{H}_0) \left[1 - \mathbf{D}^T \frac{1}{\mathcal{E} - \mathcal{H}_0} \mathbf{D} \right]. \quad (2.4)$$

Therefore, the spectrum of unstable (resonance) states is given in our model by the equation

$$1 - \mathbf{D}^T \frac{1}{\mathcal{E} - \mathcal{H}_0} \mathbf{D} = 1 - \mathbf{D}^T \mathcal{G}_0(\mathcal{E}) \mathbf{D} = 0 \quad (2.5)$$

in the complex energy plane.

The factorized structure of the external interaction (2.2) allows us to simplify the Green's matrix $\mathcal{G}_0(\mathcal{E}) = (\mathcal{E} - \mathcal{H}_0)^{-1}$ to

$$\mathcal{G}_0(\mathcal{E}) = G_0(\mathcal{E}) - \frac{i}{2} G_0(\mathcal{E}) A \frac{1}{1 + \frac{i}{2} \hat{K}_0(\mathcal{E})} A^T G_0(\mathcal{E}) \quad (2.6)$$

by subsequent iterations of the Dyson equation

$$\mathcal{G}_0(\mathcal{E}) = G_0(\mathcal{E}) - \frac{i}{2} G_0(\mathcal{E}) W \mathcal{G}_0(\mathcal{E}). \quad (2.7)$$

Here $G_0(\mathcal{E}) = (\mathcal{E} - H_0)^{-1}$ is the unperturbed Green's matrix while $\hat{K}_0(\mathcal{E}) = A^T G_0(\mathcal{E}) A$ is the corresponding K -matrix. These matrices describe the unperturbed intrinsic motion and its coupling to the continuum, respectively. Therefore they have the levels e_n as their poles in the complex energy plane.

With the help of (2.6), equation (2.5) can be presented in the form

$$1 - \mathbf{D}^T G_0(\mathcal{E}) \mathbf{D} + \frac{i}{2} \mathbf{D}^T G_0(\mathcal{E}) A \frac{1}{1 + \frac{i}{2} \hat{K}_0(\mathcal{E})} A^T G_0(\mathcal{E}) \mathbf{D} = 0. \quad (2.8)$$

The last term on the l.h.s. describes the influence of the external interaction via the continuum onto the energy spectrum of the internal motion. This equation can easily be reduced to an algebraic equation of the order N producing the N complex energies of the resonance states.

In a similar manner we obtain

$$\hat{K}(E) = A^T \frac{1}{E - H} A = \hat{K}_0(E) + A^T G_0(E) \mathbf{D} \frac{1}{1 - \mathbf{D}^T G_0(E) \mathbf{D}} \mathbf{D}^T G_0(E) A \quad (2.9)$$

for the full K -matrix. The additional term is due to the internal interaction. Eqs. (2.8) and (2.9) present the explicit solution of our model. In the following we investigate this solution analytically as well as numerically. Before finishing this section, let us consider the limit of a very strong internal interaction (see below for the explicit condition). It is well known that such an interaction leads to the creation of a collective vibration mode which is shifted in energy from the location of the original unperturbed (parental) levels by a distance of the order of magnitude of $Tr(\mathbf{D}\mathbf{D}^T) = \mathbf{D}^2$. Characterizing this location by some average position $\varepsilon_0 = \langle e \rangle$, we find

$$\mathcal{E}_{gr} = E_{gr} - \frac{i}{2} \Gamma_{gr} = \varepsilon_0 + \mathbf{D}^2 - \frac{i}{2} \hat{\mathbf{A}}_d^2 \quad (2.10)$$

from the secular equation (2.8) for the complex energy of the only isolated giant resonance state. In the energy domain of this state, the K -matrix (2.9) acquires the standard resonance form

$$\hat{K}(E) = \frac{\hat{\mathbf{A}}_d^T \hat{\mathbf{A}}_d}{E - E_{gr}} \quad (2.11)$$

where the components of the row vector $\hat{\mathbf{A}}$ are equal to the projections

$$A_d^c \equiv (\mathbf{d} \cdot \mathbf{A}^c) \quad (2.12)$$

of the decay amplitudes \mathbf{A}^c onto the unit dipole vector $\mathbf{d} = \mathbf{D}/\sqrt{\mathbf{D}^2}$. The residues

$$\Gamma_{gr}^c = (\mathbf{d} \cdot \mathbf{A}^c)^2 \quad (2.13)$$

of the diagonal elements of the K -matrix (2.11) determine the partial escape widths of the GR state.

Because of the overlapping of different resonance states, eqs. (2.8, 2.9) lead, generally, to a more complicated picture. The elements of the channel space matrix $\hat{K}(E)$ can be presented by

$$K^{cc'}(E) = \text{Tr} \left(G(E) \mathbf{A}^c (\mathbf{A}^{c'})^T \right) \quad (2.14)$$

as the matrix trace in the Hilbert space of the internal motion. Therefore, it is invariant with respect to any transformation of the basis in this space. Such transformations connect different parametrizations of the K -matrix. The concrete choice of the parametrization is dictated by physical reasons as well as by convenience. For instance, one can use the eigenbasis of the hamiltonian H to represent the K -matrix as the sum

$$\hat{K}(E) = \sum_r \frac{\hat{\mathbf{A}}_r^T \hat{\mathbf{A}}_r}{E - \varepsilon_r} \quad (2.15)$$

over all the internal eigenstates $\Phi^{(r)}$. Each term of this sum is directly analogous to the single-resonance expression (2.11). The row vectors $\hat{\mathbf{A}}_r$ consist of the real components

$$A_r^c = \Phi^{(r)} \cdot \mathbf{A}^c \quad (2.16)$$

along the eigenvectors $\Phi^{(r)}$ belonging to the eigenenergies ε_r . The residues

$$\Gamma_r^c = (A_r^c)^2 \quad (2.17)$$

at the poles of the diagonal elements of the matrix (2.15) characterize the coupling of the intrinsic eigenstates $\Phi^{(r)}$ to individual channel states. In analogy with (2.13), we will call them the partial escape widths of the r th resonance. It must be stressed however that, contrary to the case of isolated resonances, the real residues Γ_r^c differ from the generally complex residues at the poles of the scattering amplitudes when the resonances overlap. For this reason, the very concept of the partial widths becomes ambiguous when overlapping resonances are concerned (see part II [23] for more details).

3 Collective Phenomena

3.1 Internal Collectivity

It is convenient to treat the factorized internal residual interaction $\mathbf{D}\mathbf{D}^T$ formally in the same manner as the external one introducing an additional imaginary "channel" with the "decay amplitudes" $\mathbf{A}^0 \equiv \sqrt{2i}\mathbf{D}$. Defining the new matrix

$$\mathcal{A} = \begin{pmatrix} \mathbf{A}^0 & \mathbf{A}^1 & \dots & \mathbf{A}^k \end{pmatrix}, \quad (3.1)$$

we will consider the matrix

$$\hat{\mathcal{K}}(E) = \mathcal{A}^T G(E) \mathcal{A} \quad (3.2)$$

in the enlarged channel space. Besides the $k \times k$ block $\hat{K}(E)$ this matrix contains along the main diagonal additionally the function

$$\mathcal{K}^{00}(E) \equiv 2i P(E) = 2i \mathbf{D}^T G(E) \mathbf{D} \quad (3.3)$$

which carries information on the degree of collectivity of the internal motion.

The degree of collectivity of an internal eigenstate $\Phi^{(r)}$ of the hermitian part H of the effective Hamiltonian (2.3) is characterized by the residue $\left(\Phi^{(r)} \cdot \mathbf{D}\right)^2$ of the function $P(E)$ at its pole at the eigenenergy $E = \varepsilon_r$. To estimate this degree, let us note first that the collectivity can become appreciable only if the internal interaction is sufficiently strong and dominates the internal dynamics. Under such a condition it is natural to start with the diagonalization of the interaction matrix $\mathbf{D}\mathbf{D}^T$. Because of its factorized structure, this matrix possesses the only nonzero eigenvalue \mathbf{D}^2 belonging to the eigenstate

$$\Upsilon^{(1)} = \mathbf{d}. \quad (3.4)$$

The rest of the eigenstates $\Upsilon^{(\mu)}$, $\mu = 2, 3, \dots, N$ remains arbitrary because of the degeneracy of the zero eigenvalue. These states span a basis in the $(N - 1)$ -dimensional subspace orthogonal to the vector \mathbf{d} . We will fix this basis a little bit later.

In the new basis the function $P(E)$ reduces to

$$P(E) = \text{Tr} \left(\frac{1}{E - H} \mathbf{D} \mathbf{D}^T \right) = \mathbf{D}^2 G_{coll}(E) \quad (3.5)$$

where $G_{coll}(E)$ is the upper diagonal matrix element of the internal Green's matrix. The Hamiltonian matrix H looks as follows:

$$H = \begin{pmatrix} \varepsilon_0 + \mathbf{D}^2 & \mathbf{h}^T \\ \mathbf{h} & \tilde{H} \end{pmatrix}. \quad (3.6)$$

Here the energy

$$\varepsilon_0 \equiv \sum_n e_n d_n^2 = \langle e \rangle \quad (3.7)$$

is the weighted mean position of the parental levels e_n . The $(N - 1)$ -dimensional vector \mathbf{h} has the components

$$h_\mu = \sum_n e_n d_n \Upsilon_n^{(\mu)} \quad (3.8)$$

while the matrix elements of the $(N - 1) \times (N - 1)$ submatrix \tilde{H} are equal to

$$\tilde{H}_{\mu\nu} = \sum_n e_n \Upsilon_n^{(\mu)} \Upsilon_n^{(\nu)}. \quad (3.9)$$

Representing similar to (3.6) the resolvent $G(E)$ as

$$G(E) = \begin{pmatrix} G_{coll}(E) & \mathbf{F}^T(E) \\ \mathbf{F}(E) & \tilde{G}(E) \end{pmatrix}, \quad (3.10)$$

one finds

$$G_{coll}(E) = \frac{1}{E - \varepsilon_0 - \mathbf{D}^2 - \mathbf{h}^T \frac{1}{E - \tilde{H}} \mathbf{h}} \quad (3.11)$$

when

$$\mathbf{F}(E) = \frac{1}{E - \tilde{H}} \mathbf{h} G_{coll}(E) \quad (3.12)$$

and

$$\tilde{G}(E) = \frac{1}{E - \tilde{H}} + \frac{1}{E - \tilde{H}} \mathbf{h} \mathbf{h}^T \frac{1}{E - \tilde{H}} G_{coll}(E). \quad (3.13)$$

Each eigenvalue ε_r of the Hamiltonian H satisfies the equation

$$\lambda(E) \equiv E - \varepsilon_0 - \mathbf{D}^2 - \mathbf{h}^T \frac{1}{E - \tilde{H}} \mathbf{h} = 0 \quad (3.14)$$

and the value

$$f^r = \left(\mathbf{d} \cdot \Phi^{(r)} \right)^2 = \text{Res} P(\varepsilon_r) / \mathbf{D}^2 = \left(\frac{d\lambda(E)}{dE} \right)_{E=\varepsilon_r}^{-1} = \left[1 + \mathbf{h}^T \left(\frac{1}{\varepsilon_r - \tilde{H}} \right)^2 \mathbf{h} \right]^{-1}, \quad (3.15)$$

subject to the condition

$$\sum_r f^r = 1, \quad (3.16)$$

describes the part of the dipole strength carried by the eigenstate $\Phi^{(r)}$.

Further, we diagonalize the submatrix (3.9),

$$\sum_n e_n \Upsilon_n^{(\mu)} \Upsilon_n^{(\nu)} = \tilde{\varepsilon}_\mu \delta_{\mu\nu} \quad (3.17)$$

by choosing the till now unspecified basic vectors $\mathbf{\Upsilon}^{(\mu)}$ to coincide with its eigenvectors. Using the completeness condition

$$\sum_{\nu} \Upsilon_m^{(\nu)} \Upsilon_n^{(\nu)} = \delta_{mn} - d_m d_n \quad (3.18)$$

one easily obtains

$$\mathbf{\Upsilon}^{(\mu)} = -h_{\mu} \frac{1}{\tilde{\varepsilon}_{\mu} - H_0} \mathbf{d} . \quad (3.19)$$

The matrix elements h_{μ} play the role of the normalization coefficients of the eigenvectors and are equal to

$$h_{\mu} = \left[\mathbf{d}^T \left(\frac{1}{\tilde{\varepsilon}_{\mu} - H_0} \right)^2 \mathbf{d} \right]^{-\frac{1}{2}} . \quad (3.20)$$

The orthogonality condition $\mathbf{d} \cdot \mathbf{\Upsilon}^{(\mu)} = 0$ immediately leads to the equation

$$\mathbf{d}^T \frac{1}{\tilde{\varepsilon} - H_0} \mathbf{d} = \sum_n \frac{d_n^2}{\tilde{\varepsilon} - e_n} = 0 \quad (3.21)$$

for the eigenvalue spectrum of the submatrix \tilde{H} . Obviously, each eigenvalue $\tilde{\varepsilon}_{\mu}$ lies between two neighboring parental levels e . Therefore, the levels $\tilde{\varepsilon}$ are shifted, with respect to the original ones, by distances of the order of magnitude of the unperturbed mean level spacing. This is much smaller than the energy shift $\sim \mathbf{D}^2$ of the collective level in the upper corner of the Hamiltonian matrix (3.6).

The collective level $\varepsilon_{coll} = \varepsilon_0 + \mathbf{D}^2$ is still mixed with the $N - 1$ levels $\tilde{\varepsilon}_{\mu}$ via the matrix elements h_{μ} . The square length of the vector \mathbf{h} turns out to be equal to the variance

$$\mathbf{h}^2 = \sum_n e_n^2 d_n^2 - \left(\sum_n e_n d_n^2 \right)^2 = \sum_n (e_n - \langle e \rangle)^2 d_n^2 = \langle (e - \langle e \rangle)^2 \rangle = \Delta_e^2 \quad (3.22)$$

of the distribution of the parental levels. This leads to the estimation $|h_{\mu}| \sim \Delta_e / \sqrt{N - 1}$ of the individual matrix elements. Therefore, the mixing is governed by the parameter $\kappa = \Delta_e / \mathbf{D}^2$. Suggesting that this parameter is small, $\kappa \ll 1$, one can use the standard perturbation expansion which gives

$$\varepsilon_1 = \varepsilon_0 + (1 + \kappa^2) \mathbf{D}^2 , \quad |\varepsilon_r - \tilde{\varepsilon}_r| \sim \frac{\kappa^2}{N - 1} \mathbf{D}^2 \quad (r \neq 1) \quad (3.23)$$

for the levels and

$$f^1 = 1 - \kappa^2 , \quad f^r \sim \frac{\kappa^2}{N - 1} \quad (r \neq 1) \quad (3.24)$$

for the dipole strengths. The first level accumulates the lion's share of both the total dipole strength and the energy displacement. In the limit $\kappa \rightarrow 0$ the collectivity of the first level becomes perfect while the rest of the levels carries no collectivity at all.

3.2 External Collectivity

Let us now turn to the properties of the K -matrix eq. (2.9). The strong internal interaction causes a remarkable redistribution of the original residues $\Gamma_n^c = (A_n^c)^2$. In this case, the Υ -basis (3.4, 3.19) constructed above becomes a preferential one. Taking into account eqs. (3.12, 3.13) one gets in this basis

$$\hat{K}(E) = \left[\hat{\mathbf{A}}_d + \mathbf{h}^T \frac{1}{E - \tilde{H}} A_{\perp} \right]^T \left[\hat{\mathbf{A}}_d + \mathbf{h}^T \frac{1}{E - \tilde{H}} A_{\perp} \right] G_{coll}(E) + A_{\perp}^T \frac{1}{E - \tilde{H}} A_{\perp} . \quad (3.25)$$

The rectangular submatrix A_{\perp} is composed of $(N - 1)$ -dimensional column vectors \mathbf{A}_{\perp}^c orthogonal to the dipole vector,

$$(\mathbf{d} \cdot \mathbf{A}_{\perp}^c) = 0 , \quad A_{\mu}^c = \left(\mathbf{\Upsilon}^{(\mu)} \cdot \mathbf{A}^c \right) , \quad (3.26)$$

whereas the row vector $\hat{\mathbf{A}}_d$ of the longitudinal components A_d^c is defined in (2.12). It can easily be checked that the contributions of the poles at the energies $E = \tilde{\varepsilon}_\mu$ in the two terms on the r.h.s. perfectly cancel each other. The actual poles of the K -matrix are given by the roots ε_r of the equation (3.14).

It immediately follows from (3.25) that the partial widths (2.17) are equal to

$$\Gamma_r^c = f^r \left[A_d^c + \mathbf{h}^T \frac{1}{\varepsilon_r - \tilde{H}} \mathbf{A}_\perp^c \right]^2 \quad (3.27)$$

and depend on the relative strength κ of the residual mixing. In particular, by using condition (3.18), one finds

$$\left| \mathbf{h}^T \frac{1}{\varepsilon_1 - \tilde{H}} \mathbf{A}_\perp^c \right| \approx \frac{1}{\mathbf{D}^2} |(\mathbf{h} \cdot \mathbf{A}_\perp^c)| \approx \frac{1}{\mathbf{D}^2} \left| \sum_n (e_n - \varepsilon_0) d_n (A_\perp^c)_n \right| \leq \kappa \sqrt{(\mathbf{A}_\perp^c)^2} \quad (3.28)$$

for the collective level if $\kappa \ll 1$.

In the square bracket of eq. (3.27) the first term dominates for the collective level $r = 1$ as long as $|A_d^c|/|A_\perp^c| \gg \kappa$. Therefore, $\Gamma_1^c \approx (A_d^c)^2$ under such a condition. The remaining part $(\mathbf{A}_\perp^c)^2$ is distributed over the $N - 1$ levels lying in the energy interval $\sim \Delta_e$ around the point ε_0 . In this region the pattern turns out to depend crucially on the ratio $(\mathbf{A}_\perp^c)^2/\Delta_e^2$. Each state acquires the partial width $\sim (\mathbf{A}_\perp^c)^2/(N - 1)$ if this ratio is small while a strong redistribution of the widths occurs in the opposite case $(\mathbf{A}_\perp^c)^2/\Delta_e^2 \gg 1$. It is called "width collectivization" [31, 28, 32] or "trapping effect" [33, 34, 35, 36, 37, 38]: k eigenstates $\Phi^{(r)}$ get large components along the vector \mathbf{A}_\perp^c and accumulate almost the total value $(\mathbf{A}_\perp^c)^2$ (see also [39, 40, 41]). This phenomenon was first observed in realistic numerical simulations of nuclear reactions in [42, 43, 44].

In the limit $\kappa = 0$, when the internal collectivity is maximal, the expression (3.25) reduces to

$$\hat{K}(E) = \frac{\hat{\mathbf{A}}_d^T \hat{\mathbf{A}}_d}{E - \varepsilon_{coll}} + \frac{\hat{X}_\perp}{E - \varepsilon_0}. \quad (3.29)$$

Here the matrix

$$\hat{X}_\perp = \hat{X} - \hat{\mathbf{A}}_d^T \hat{\mathbf{A}}_d = A_\perp^T A_\perp \quad (3.30)$$

is composed of the scalar products $(\mathbf{A}_\perp^c \cdot \mathbf{A}_\perp^{c'})$ in the orthogonal subspace when the matrix

$$\hat{X} = A^T A \quad (3.31)$$

is formed by the scalar products $(\mathbf{A}^{(c)} \cdot \mathbf{A}^{(c')})$ [30, 31] in the full Hilbert space.

One can immediately see from eq. (3.29) that the partial widths (2.17) of the only collective state with the energy $\varepsilon_{coll} = \varepsilon_0 + \mathbf{D}^2$ are equal to:

$$\Gamma_{coll}^c = (A_d^c)^2 \quad (3.32)$$

in agreement with eq. (2.13).

In the given limit, the other states are jointly presented in (3.29) by the single pole at the energy $E = \varepsilon_0$. The residues at this pole do not factorize contrary to the residue at the pole of the collective part. This means that different linear superpositions of the original states are excited via different channels at the same energy ε_0 . To find the corresponding partial widths Γ_r^c , (2.17), one has first to diagonalize the matrix \hat{X}_\perp . Then

$$\Gamma_r^c = \gamma_\perp^r \left(\xi_{\perp c}^{(r)} \right)^2, \quad (r = 1, 2, \dots, k) \quad (3.33)$$

is expressed in terms of the eigenvalues γ_\perp^r and the left eigenvectors $\xi_{\perp}^{(r)}$ of the matrix \hat{X}_\perp . One sees that in the considered limit of very strong internal collectivity only k superpositions out of the $N - 1$ ones with the energy ε_0 possess nonzero partial width (2.17). They absorb the part

$$\sum_{r=1}^{r=k} \Gamma_r^c = (\mathbf{A}_\perp^c)^2 = (\mathbf{A}^c)^2 - \Gamma_{coll}^c \quad (3.34)$$

of the total original value $(\mathbf{A}^c)^2$. We conclude that the partial widths Γ_r^c of the $k + 1$ states presented in the K -matrix (3.29) are formed by contributions of all parental states.

4 Interplay of Two Kinds of Collectivity. Interference of Doorway Resonances

4.1 The Doorway Basis

Now we turn to the resonance spectrum resulting from the interplay and competition of both kinds of collectivity. Since the interaction plays the dominant role in the dynamics studied, we start with the consideration of the interaction matrix

$$\mathcal{H}^{(int)} = \mathbf{D}\mathbf{D}^T - \frac{i}{2}\mathbf{A}\mathbf{A}^T. \quad (4.1)$$

The manifold of the $k+1$ linearly independent vectors \mathbf{D} and \mathbf{A}^c forms a $(k+1)$ -dimensional subspace in the internal Hilbert space the total dimension of which is N . It is convenient to choose the first $k+1$ basis vectors of the total Hilbert space in such a manner that they belong entirely to this subspace. Then only the upper left $(k+1) \times (k+1)$ block of the interaction matrix will contain non-zero matrix elements. We proceed in the following three steps:

(i) Let us first orthogonalize the set of the k vectors \mathbf{A}^c . For this purpose, we diagonalize the matrix \hat{X} , eq. (3.31), of the scalar products of these vectors. Let $\hat{\xi}$ be the matrix of the (left) eigenvectors,

$$\hat{\xi} \hat{X} = \hat{\gamma} \hat{\xi} \quad (4.2)$$

where

$$\hat{\gamma} = \text{diag}(\gamma^1 \gamma^2 \dots \gamma^k) \quad (4.3)$$

is the diagonal matrix of the eigenvalues. It is then obvious that the rectangular matrix

$$\mathbf{a} = \mathbf{A} \hat{\xi}^T \gamma^{-\frac{1}{2}} \quad (4.4)$$

consists of k mutually orthogonal unit vectors

$$\mathbf{a}^c = \frac{1}{\sqrt{\gamma^c}} \sum_{c'} \xi_{c'}^{(c)} \mathbf{A}^{c'}. \quad (4.5)$$

Adding to this set an extra unit vector \mathbf{a}^0 which is orthogonal to all of them, one obtains a new basis in the non-trivial part of the total Hilbert space. One can easily see that the vectors $\mathbf{a}^0, \mathbf{a}^c$ are just the eigenvectors of the antihermitian part $W = \mathbf{A}\mathbf{A}^T$. Therefore, this matrix becomes diagonal,

$$W = \text{diag}(0 \ \hat{\gamma}). \quad (4.6)$$

Its nonzero eigenvalues γ^c coincide with those of the matrix \hat{X} [31, 28]. For the present, we drop the matrix blocks and the vector components which belong to the complementary $(N-(k+1))$ -dimensional subspace and consist of zero elements.

In the chosen basis the unit dipole vector \mathbf{d} has the components

$$d_0 = (\mathbf{a}^0 \cdot \mathbf{d}) = \sin\Theta, \quad d_c = (\mathbf{a}^c \cdot \mathbf{d}) = \cos\Theta \cos\varphi_c; \\ \sum_c \cos^2\varphi_c = 1. \quad (4.7)$$

Here we have introduced the angle $\Theta, (0 \leq \Theta \leq \pi/2)$, between the dipole vector \mathbf{D} and the k -dimensional subspace spanned by the decay vectors \mathbf{A}^c . This angle is an important parameter which governs the interference effects under consideration.

The matrix of the internal interaction reads

$$\mathbf{D}^2 \begin{pmatrix} \sin^2\Theta & \sin\Theta \cos\Theta \mathbf{l}^T \\ \sin\Theta \cos\Theta \mathbf{l} & \cos^2\Theta \mathbf{l}\mathbf{l}^T \end{pmatrix} \quad (4.8)$$

where \mathbf{l} stands for the unit vector with the components $l_c = \cos\varphi_c$.

(ii) Next we rotate the considered $(k+1)$ -dimensional subspace around the unit vector \mathbf{a}^0 to put the unit dipole vector \mathbf{d} into the coordinate plane $(0,1)$. This is done by the diagonalization of the $k \times k$ submatrix $\mathbf{l}\mathbf{l}^T$ with the help of a k -dimensional orthogonal matrix $\hat{\eta}$ the first column of which coincides with the vector \mathbf{l} . This transformation resembles that described in subsection 4.1.

Now only two nonzero components of the unit dipole vector \mathbf{d} are left which are equal to

$$d_0 = \sin\Theta, \quad d_1 = \cos\Theta, \quad (4.9)$$

and only the 2×2 upper block of the internal interaction matrix (4.8) remains non-trivial. The vectors \mathbf{A}^c are transformed into

$$A_0^c = 0, \quad A_1^c = \sum_{c'} \sqrt{\gamma^{c'}} \cos\varphi_{c'} \xi_c^{(c')}, \quad A_\alpha^c = \sum_{c'} \sqrt{\gamma^{c'}} \eta_{c'}^{(\alpha)} \xi_c^{(c')}, \quad (4.10)$$

so that

$$A_d^c = \cos\Theta A_1^c = \cos\Theta \sum_{c'} \sqrt{\gamma^{c'}} \cos\varphi_{c'} \xi_c^{(c')} \quad (4.11)$$

when the lower diagonal submatrix $\hat{\gamma}$ in eq. (4.6) is replaced by

$$\hat{\gamma} \rightarrow \begin{pmatrix} \langle \gamma \rangle & \mathbf{w}^T \\ \mathbf{w} & \tilde{W} \end{pmatrix}. \quad (4.12)$$

Here

$$\langle \gamma \rangle = \sum_c (A_1^c)^2 = \sum_c \gamma^c \cos^2\varphi_c, \quad w^{(\alpha)} = \sum_c \gamma^c \cos\varphi_c \eta_c^{(\alpha)}, \quad (\alpha = 2, 3, \dots, k) \quad (4.13)$$

and

$$W_{\alpha\alpha'} = \sum_c \gamma^c \eta_c^{(\alpha)} \eta_c^{(\alpha')}. \quad (4.14)$$

(iii). Returning now into the total N -dimensional Hilbert space, the two consecutive transformations just described are part of the global transformation produced by the orthogonal matrix

$$\Omega = \left(\Omega^{(0)} = \mathbf{a}^{(0)} \quad \Omega^{(1)} = \sum_c \cos\varphi_c \mathbf{a}^{(c)} \quad \Omega^{(2 \leq \alpha \leq k)} = \sum_c \eta_c^{(\alpha)} \mathbf{a}^{(c)} \quad \Omega^{(k+1 \leq s \leq N-1)} \right). \quad (4.15)$$

In (4.15), the two groups of vectors, the $N - (k+1)$ vectors $\Omega^{(k+1 \leq s \leq N-1)}$ in the full space and the $k-1$ ones $\Omega^{(2 \leq s \leq k)}$ in the $k+1$ -dimensional subspace, can still be chosen arbitrarily. We will fix them later.

In the Ω -basis, the diagonal matrix elements of the unperturbed Hamiltonian H_0 are given by the weighted mean positions

$$\tilde{e}_s = \sum_n e_n \left(\Omega_n^{(s)} \right)^2, \quad (s = 0, 1, \dots, N-1) \quad (4.16)$$

when the off-diagonal elements

$$V_{ss'} = \sum_n e_n \Omega_n^{(s)} \Omega_n^{(s')}, \quad (s \neq s') \quad (4.17)$$

obey the general sum rules

$$\sum_{s' \neq s} V_{ss'}^2 = \sum_{s'} (e_{s'} - \tilde{e}_s)^2 \left(\Omega_n^{(s)} \right)^2 \sim \Delta_e^2 \quad (4.18)$$

(compare with eq. (3.22)). Since we do not expect any special relation between the original basis and the doorway one, all off-diagonal matrix elements are suggested to be of the same order of magnitude. This leads to the estimation

$$|V_{s \neq s'}| \sim \Delta_e / \sqrt{N-1} \quad (4.19)$$

similar to that found in subsection 4.1.

We now use the last $N - (k + 1)$ vectors $\mathbf{\Omega}^{(s)}$, ($s \equiv tr = k + 1, k + 2, \dots, N - 1$) in (4.15) in order to diagonalize the lower block of the unperturbed Hamiltonian [28, 32],

$$\sum_n e_n \Omega_n^{(tr)} \Omega_n^{(tr')} = \tilde{\varepsilon}_{tr} \delta_{tr tr'} . \quad (4.20)$$

In the new picture, the $N_{tr} = N - k - 1$ eigenstates with the energies $\tilde{\varepsilon}_{tr}$ (which lie within the original energy region Δ_e [32]) are "trapped" [44, 31, 39], i.e. they do not have a direct access to the continuum. These states can decay only via the first $N_{dw} = k + 1$ "doorway" states to which they are coupled by the hermitian residual interaction

$$V_{dw tr} = \sum_n e_n \Omega_n^{(dw)} \Omega_n^{(tr)} . \quad (4.21)$$

This interaction appears from the initial unperturbed hamiltonian being transformed into the doorway basis.

A typical value for the widths of the doorway states is $\langle \gamma \rangle \approx \frac{1}{k} Tr W \sim \langle (\mathbf{A}^c)^2 \rangle$. Only one of them can, under certain conditions, become almost stable (see subsection 5.2) but then it is displaced by a distance $\sim \mathbf{D}^2$. Therefore, all trapped states lie in the complex energy plane far from the N_{dw} doorway states and their admixture to the latter is small as one of the ratios

$$\kappa = \frac{\Delta_e}{\mathbf{D}^2} , \quad \kappa' = \frac{\Delta_e}{\langle \gamma \rangle} . \quad (4.22)$$

According to the estimation (4.19) the trapped states acquire the widths $\sim \frac{N_{dw}}{N_{tr}} \kappa'^2 \langle \gamma \rangle$. The energy shifts of the trapped states are of the same order of magnitude. These states are responsible therefore for the fine structure effects (with the characteristic energy scale $\Delta_e/(N_{tr})$) in the energy domain of the parental levels. They become irrelevant when $(\kappa, \kappa') \rightarrow 0$.

The doorway $N_{dw} \times N_{dw}$ part of the effective Hamiltonian

$$\mathcal{H}^{(dw)} = \begin{pmatrix} \mathcal{H}^{(coll)} & \chi^T \\ \chi & \tilde{\mathcal{H}} \end{pmatrix} \quad (4.23)$$

includes two different blocks along the main diagonal. Only the upper 2×2 block

$$\mathcal{H}^{(coll)} = \begin{pmatrix} \tilde{e}_0 + \sin^2 \Theta \mathbf{D}^2 & V_{01} + \sin \Theta \cos \Theta \mathbf{D}^2 \\ V_{10} + \sin \Theta \cos \Theta \mathbf{D}^2 & \tilde{e}_1 + \cos^2 \Theta \mathbf{D}^2 \end{pmatrix} - \frac{i}{2} \langle \gamma \rangle \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (4.24)$$

contains, along with the collectivity via the continuum, the collective effects induced by the internal residual interaction. Due to the mixing described by the off-diagonal matrix elements, the widths, energy shifts and dipole strengths of the two eigenstates of this block are generally comparable to each other provided that the angle Θ differs from 0 and $\pi/2$ so that $\sin^2 \Theta \sim \cos^2 \Theta$.

The $(k - 1) \times (k - 1)$ block

$$\tilde{\mathcal{H}}_{\alpha \alpha'} = \sum_{c, c'} \eta_c^{(\alpha)} \left[\sum_n e_n a_n^{(c)} a_n^{(c')} - \frac{i}{2} \gamma^c \delta_{c c'} \right] \eta_{c'}^{(\alpha')} \quad (4.25)$$

(see eq. (4.14) for the antihermitian part) describes the states without both dipole strengths (according to (4.9)) and collective energy shifts. Generally, they are strongly mixed with each other. The set of vectors $\eta^{(\alpha)}$ can be used to diagonalize either the hermitian or the antihermitian part of $\tilde{\mathcal{H}}$ depending on which of them dominates the dynamics inside this block. The remaining part may then be treated as a weak perturbation. However when both, the hermitian and antihermitian parts, are of equal importance the full Hamiltonian $\tilde{\mathcal{H}}$ must be diagonalized.

The two doorway blocks just described are coupled by the complex interaction

$$\chi = \left(\mathbf{v}^{(0)} \mathbf{v}^{(1)} \right) - \frac{i}{2} (\mathbf{0} \mathbf{w}) ; \quad (4.26)$$

$$v_{\alpha}^{(0)} \equiv V_{0\alpha} , \quad v_{\alpha}^{(1)} \equiv V_{1\alpha} , \quad w^{(\alpha)} = \sum_c \gamma^c \cos \varphi_c \eta_c^{(\alpha)} . \quad (4.27)$$

The influence of the hermitian part is weak as κ^2, κ'^2 again. The strength of the antihermitian coupling can be estimated by using the identity

$$\mathbf{w}^2 \equiv \sum_{\alpha} \left(w^{(\alpha)} \right)^2 = \sum_c (\gamma^c)^2 \cos^2 \varphi_c - \left(\sum_c \gamma^c \cos^2 \varphi_c \right)^2 = \Delta_{\gamma}^2 \quad (4.28)$$

which is the counterpart of the eq. (3.22). In the limit $\kappa^2, \kappa'^2 \rightarrow 0$, the two blocks of doorway states with and without internal collectivity are mixed only due to the antihermitian interaction via the continuum. The strength of this interaction is determined by the variance of the nonzero eigenvalues of the antihermitian part W , eq. (2.2).

4.2 Resonance Spectrum and Dipole Strengths of Doorway States

The interaction W of the intrinsic states via the continuum causes a strong redistribution of the dipole strength when the doorway states overlap. In this case, the dipole strengths of the decaying states rather than those of the intrinsic eigenvectors $\Phi^{(r)}$ (as defined in (3.15)) should be used to characterize electromagnetic properties of the open system considered. The natural and appropriate extension of the definition (3.15) to the dipole strength of an eigenvector $\Psi^{(s)}$ of the total nonhermitian effective Hamiltonian (2.3) is given by

$$\tilde{f}^s = \frac{1}{U_s} \left| \mathbf{d} \cdot \Psi^{(s)} \right|^2 \quad (4.29)$$

where $U_s = \left(\Psi^{(s)*} \cdot \Psi^{(s)} \right)$. It is in accordance with the definition of expectation values if the Hamiltonian is non-hermitean and the eigenfunctions form a bi-orthogonal system [13].

The quantities (4.29) are directly linked with the resonance spectrum. Multiplying the equation

$$\mathcal{H}\Psi = \Psi\mathcal{E} \quad (4.30)$$

for the matrix Ψ of the unstable eigenstates $\Psi^{(s)}$ by the matrix Ψ^{\dagger} from the left side and adding the hermitian conjugate of the relation thus received, one obtains

$$\tilde{f}^s = \frac{1}{\mathbf{D}^2} \left(E_s - \sum_n e_n \frac{|\Psi_n^{(s)}|^2}{U_s} \right) \quad (4.31)$$

where E_s is the resonance energy.

In the doorway basis introduced in subsection 5.1, the upper $N_{dw} \times N_{dw}$ block of the effective Hamiltonian is totally decoupled from the lower block of the trapped states if one neglects the small matrix elements (4.21) the contributions of which are of the order of magnitude κ^2 or κ'^2 . Omitting them, one neglects the fine structure variations of the transition amplitudes in the energy domain of the parental levels as mentioned above (sec. 5.1; see also part II). In such an approximation, the trapped states remain stable and are entirely excluded from all further calculations. Then relation (4.31) becomes especially simple

$$\tilde{f}^{dw} = \frac{1}{\mathbf{D}^2} (E_{dw} - \varepsilon_0) . \quad (4.32)$$

Taking into account eq. (4.23), the secular equation (2.5) can now be reduced to

$$\text{Det} \left(\mathcal{E} - \mathcal{H}^{(coll)} - \mathcal{Q}(\mathcal{E}) \right) = 0 . \quad (4.33)$$

The second order self-energy matrix

$$\mathcal{Q}(\mathcal{E}) = \chi^T \frac{1}{\mathcal{E} - \tilde{\mathcal{H}}} \chi \quad (4.34)$$

describes the virtual transitions between the two types of doorway states. Its explicit form depends on the interference regime inside the second group. We further assume that the antihermitian part dominates in the Hamiltonian submatrix (4.25). Therefore we diagonalize first this part by demanding the vectors $\eta^{(\alpha)}$ to satisfy the conditions

$$W_{\alpha\alpha'} = \sum_c \gamma^c \eta_c^{(\alpha)} \eta_c^{(\alpha')} = \tilde{\gamma}^\alpha \delta_{\alpha\alpha'} . \quad (4.35)$$

The opposite case with dominating hermitian part can be treated in an analogous manner.

Going further along the same line as in subsection 4.1, one finds for the eigenvectors

$$\eta_c^{(\alpha)} = w^{(\alpha)} \frac{\cos \varphi_c}{\gamma^c - \tilde{\gamma}^\alpha} \quad (4.36)$$

with the normalization condition

$$w^{(\alpha)} = \left[\sum_c \frac{\cos^2 \varphi_c}{(\gamma^c - \tilde{\gamma}^\alpha)^2} \right]^{-\frac{1}{2}} . \quad (4.37)$$

The corresponding eigenvalues $\tilde{\gamma}^\alpha$ are the roots of the equation

$$\sum_c \frac{\cos^2 \varphi_c}{\gamma^c - \tilde{\gamma}} = 0 . \quad (4.38)$$

Each of the $k - 1$ eigenvalues $\tilde{\gamma}^\alpha$ lies between two neighbouring values γ^c . The last three equations should be compared with eqs. (3.19) - (3.21).

Considering the hermitian part of the hamiltonian (4.25) to be a weak perturbation, one obtains

$$\tilde{\mathcal{H}}_{\alpha\alpha'} \approx \left(\tilde{e}_\alpha - \frac{i}{2} \tilde{\gamma}^\alpha \right) \delta_{\alpha\alpha'} = \tilde{\mathcal{E}}_\alpha \delta_{\alpha\alpha'} \quad (4.39)$$

in first approximation. The corrections are proportional to the ratio $\Delta_e^2/\Delta_\gamma^2$ of the variances of the unperturbed levels e_n and of the collective widths γ^c . The approximation is justified when this ratio is small.

Under the last condition, one can also neglect the hermitian part of the coupling matrix χ , eq. (4.26). The only nonzero matrix element in the right lower corner of the self-energy matrix \mathcal{Q} reads then

$$\mathcal{Q}_{11}(E) = -\frac{1}{4} \sum_\alpha \frac{w^{(\alpha)^2}}{E - \tilde{\mathcal{E}}_\alpha} \equiv -\frac{1}{4} q(E) \quad (4.40)$$

and the secular equation (4.33) reduces to

$$\Lambda(\mathcal{E}) \equiv (\mathcal{E} - \varepsilon_0) (\mathcal{E} - \varepsilon_{coll}) + \frac{i}{2} \omega(\mathcal{E}) \left(\mathcal{E} - \varepsilon_0 - \sin^2 \Theta \mathbf{D}^2 \right) = 0 \quad (4.41)$$

where the notation

$$\omega(\mathcal{E}) = \langle \gamma \rangle - \frac{i}{2} q(\mathcal{E}) \quad (4.42)$$

has been introduced. Here we neglected the matrix elements $V_{01} = V_{10}$ and set $\tilde{e}_0 = \tilde{e}_1 = \varepsilon_0$. The corresponding corrections are again proportional to κ^2, κ'^2 . The equation (4.41) is equivalent to an algebraic equation of $(k + 1)$ th order. It determines the complex energies of the $k + 1$ doorway resonances.

4.3 The Two-Level Approximation

Let us temporarily omit also the second term $q(E)$ in eq. (4.42). Then the secular equation (4.41) reduces to the same quadratic one which appears in the single-channel problem investigated in [19]. In this approximation, one is left with two interfering collective levels only. The latter problem can be easily solved exactly. (See for example [29] and [45] where different aspects of the problem are treated. It has much in common with the physics of the text-book systems of the neutral kaons [26, 46], the ρ and ω mesons [47] or the 2^+ doublet in ^8Be [27, 48].) Using the notation

$$z = \frac{\mathcal{E} - \varepsilon_0}{\mathbf{D}^2}, \quad (4.43)$$

one obtains explicitly

$$z_0 = \frac{1}{2}(1 - |x|) - \frac{i}{2} \lambda \frac{1}{2}(1 - |y|), \quad z_1 = \frac{1}{2}(1 + |x|) - \frac{i}{2} \lambda \frac{1}{2}(1 + |y|) \quad (4.44)$$

where

$$|x| = \frac{1}{\sqrt{2}} \left[\sqrt{\left(1 - \frac{1}{4}\lambda^2\right)^2 + \lambda^2 \cos^2 2\Theta} + \left(1 - \frac{1}{4}\lambda^2\right) \right]^{\frac{1}{2}} \leq 1 \quad (4.45)$$

and

$$|y| = \frac{1}{\sqrt{2}} \left[\sqrt{\left(1 - \frac{4}{\lambda^2}\right)^2 + 4 \frac{4}{\lambda^2} \cos^2 2\Theta} + \left(1 - \frac{4}{\lambda^2}\right) \right]^{\frac{1}{2}} \leq 1. \quad (4.46)$$

Apart from the angle Θ , the interference of the collective states depends on the ratio

$$\lambda \equiv \frac{\langle \gamma \rangle}{\mathbf{D}^2} \quad (4.47)$$

of the strengths of the external and internal interactions. The solution (4.44) is valid when $0 < \Theta < \pi/4$; for $\pi/4 < \Theta < \pi/2$ the imaginary parts of the two roots are to be replaced by each other. To be definite, we consider the first possibility below.

The quantity $|x|$ measures the energy distance between the two resonances,

$$E_1 - E_0 = |x| \mathbf{D}^2, \quad (4.48)$$

whereas $|y|$ measures the difference of their total widths,

$$|\Gamma_1 - \Gamma_0| = |y| \langle \gamma \rangle. \quad (4.49)$$

According to (4.32, 4.45), one further has

$$\tilde{f}^{0,1} = \frac{1}{2}(1 \mp |x|) = \text{Re } z_{0,1} \quad (4.50)$$

in the same approximation. The latter expression shows that the closer the resonances are to each other the more similar are their dipole strengths.

The situation is especially simple for the angle $\Theta = \pi/4$. In this case

$$|x| = \begin{cases} \sqrt{1 - \lambda^2/4}, & \lambda < 2 \\ 0, & \lambda > 2 \end{cases} ; \quad |y| = \begin{cases} 0, & \lambda < 2 \\ \sqrt{1 - 4/\lambda^2}, & \lambda > 2 \end{cases}. \quad (4.51)$$

In the limit $\lambda \ll 2$, the two collective levels $dw = 0$ and $dw = 1$ are separated by a large distance $\sim \mathbf{D}^2$ but have the same widths. The level $dw = 1$ carries the whole dipole strength. With growing λ the levels are getting closer and finally merge when λ reaches the value 2. For $\lambda > 2$, the dipole strengths

as well as the energies of both resonances remain equal to each other while their widths differ more and more with increasing λ .

The transition at the point $\lambda = 2$ gets smoother for other values of the angle Θ but still exists as long as Θ is not too close to 0 or $\pi/2$. If the internal interaction prevails and $\lambda \ll 2$, both collective levels have comparable widths,

$$\mathcal{E}_0 = \varepsilon_0 - \frac{i}{2} \sin^2 \Theta \langle \gamma \rangle, \quad \mathcal{E}_1 \equiv \mathcal{E}_{gr} = \varepsilon_0 + \mathbf{D}^2 - \frac{i}{2} \cos^2 \Theta \langle \gamma \rangle, \quad (4.52)$$

while only the second one is displaced by the distance \mathbf{D}^2 and carries the whole dipole strength. According to eqs. (4.9, 4.10), the total width of this level is equal to

$$\Gamma_1 = \cos^2 \Theta \langle \gamma \rangle = \hat{\mathbf{A}}_d^2 = \Gamma_{gr} \quad (4.53)$$

in full agreement with eq. (2.10). In the opposite case of the dominating external coupling, $\lambda \gg 2$, the energy displacement \mathbf{D}^2 is shared by the two collective resonances,

$$\mathcal{E}_0 = \varepsilon_0 + \sin^2 \Theta \mathbf{D}^2 - \frac{i}{2} \langle \gamma \rangle \frac{1}{\lambda^2} \sin^2 2\Theta, \quad \mathcal{E}_1 = \varepsilon_0 + \cos^2 \Theta \mathbf{D}^2 - \frac{i}{2} \langle \gamma \rangle \left(1 - \frac{1}{\lambda^2} \sin^2 2\Theta \right). \quad (4.54)$$

(Here we omitted the small corrections $\sim \lambda^{-2}$ to the positions of the resonances). The corresponding dipole strengths are, in this case, equal to

$$\tilde{f}^0 \approx \sin^2 \Theta, \quad \tilde{f}^1 \approx \cos^2 \Theta. \quad (4.55)$$

The nucleon width of the level $dw = 0$ decreases with growing λ . Finally this resonance practically disappears from the decay spectrum in the particle channels. In agreement with eq. (4.55) it gets however a nonvanishing radiation width and contributes in the photoemission process (see part II).

Let us now consider the role of the other doorway resonances $\tilde{\mathcal{E}}_\alpha \approx \varepsilon_0 - \frac{i}{2} \langle \gamma \rangle$, eq. (4.39). Substituting the complex energies $\mathcal{E}_{0,1}$ found above into the sum (4.40), one sees that the denominators of the terms of this sum contain, as a rule, one of the large quantities \mathbf{D}^2 or $\langle \gamma \rangle$. According to eq. (4.28), such terms are of the order of magnitude τ^2/k or τ'^2/k where the parameters

$$\tau = \frac{\Delta_\gamma}{\mathbf{D}^2}, \quad \tau' = \frac{\Delta_\gamma}{\langle \gamma \rangle} \quad (4.56)$$

may be expected to be reasonably small. Therefore, these doorway states acquire a relatively small dipole strength. The interference of the first two collective states remains most important and the above two-resonance approximation gives a description which is at least qualitatively satisfactory. When, however, some of the levels $\tilde{\mathcal{E}}_\alpha$ fall by chance anomalously close to one of the former two, these doorway states take part in the interference and the picture becomes more complicated.

5 Numerical Results and Discussion

To simplify the analytical study, we restricted ourselves in the foregoing sections to the case of a very strong interaction $\mathcal{H}^{(int)}$ (4.1) so that the energy range Δ_e of the unperturbed levels e_n could be neglected. In such an approximation, the $N_{tr} = N - k - 1$ trapped states are almost fully decoupled from the continuum and only the $N_{dw} = k + 1$ collective doorway states remain relevant. In this limit, the resulting picture is determined essentially by the ratio $\lambda = \langle \gamma \rangle / \mathbf{D}^2$ of the strengths of external and internal interactions and by the angle Θ between the dipole vector \mathbf{D} and the k -dimensional subspace of decay vectors \mathbf{A}^c .

Assuming further that all the vectors \mathbf{A}^c are pairwise orthogonal and have the same lengths, the matrix \hat{X} of the scalar products becomes proportional to unity and the problem is reduced exactly to the case considered in subsection 4.3 with only two decaying states which are mixed and share the total dipole strength. The physics arising from the interference of two resonance states is governed

by the effect of avoided resonance crossing. According to eq. (4.32), the dipole strengths of the two resonances behave very much like their positions in energy when considered as functions of λ . While the sums of the two resonance energies and of the two strengths remain constant, the corresponding differences decrease as functions of increasing λ up to certain minimum values which depend on the angle Θ only. The widths of the two states increase first with increasing λ but bifurcate for large λ .

The results of this two level approximation are illustrated in Fig.1. The angle Θ is chosen to give $\cos^2\Theta \approx 0.65$ and ε_0 is set to zero. The energies of the two collective resonances (measured in units of the total energy displacement \mathbf{D}^2) and their dipole strengths plotted in dependence on λ in Fig. 1(a) coincide perfectly. Fig.1(b) displays the behaviour of the widths of the two resonance states versus their energies and/or dipole strengths when λ changes in the interval $0 \div 5$.

In the following, we lift the above mentioned simplifications and check the relevance of the obtained analytical results by performing numerical calculations under less restrictive assumptions. We have chosen $N = 10$ levels e_n distributed more or less homogeneously and coupled to $k = 3$ open particle decay channels. The extension of the parental spectrum of the N discrete levels e_n is from -0.2 to 0.2 in relative units of the total energy displacement \mathbf{D}^2 . This implies that $\kappa \equiv \Delta_e/\mathbf{D}^2 \approx 0.4$. As in Fig. 1, we set $\Theta \approx 36.3^\circ$ but the lengths of the vectors \mathbf{A}^c differ from one another within 10%. The angles $\theta_{cc'}$ between the pairs \mathbf{A}^c and $\mathbf{A}^{c'}$ are confined to $0.17 \leq |\cos \theta_{cc'}| \leq 0.31$.

In Fig.2, the energies and dipole strengths of all 10 resonances are plotted as a function of λ while the changes of their total widths with λ are shown in the representation of the Γ_s versus the positions E_s and dipole strengths \tilde{f}^s , respectively. For small λ , there is only one displaced state the dipole strength of which is very close to unity. With λ increasing, first $N_{dw} = 4$ doorway states appear three of which are formed according to the three open decay channels from the group of $N - 1$ states lying around $E = 0$, while the fourth state with large dipole strength lies at the energy $E/\mathbf{D}^2 \approx 1$. These four states almost exhaust the whole sum of widths $Tr W$ and the total dipole strength while the internal as well as external collectivity of the $N_{tr} = 6$ trapped states remain small. Fig. 2(b) shows that the total dipole strength is distributed mainly over two states: the original dipole state and one out of the group around $E = 0$. It must be noted that while in the limit $\kappa = 0$ the sum rule $\sum_s \tilde{f}^s = 1$ for the dipole strengths defined as in eqs. (4.29,4.31) is fulfilled rigorously, this is not exactly the case in general. Indeed, the sum $\sum_s \tilde{\varepsilon}_s$ of all mean positions

$$\tilde{\varepsilon}_s = \sum_n \frac{|\Psi_n^{(s)}|^2}{U_s} e_n$$

of the parental levels can slightly differ from $Tr H_0 = \sum_n e_n$ which leads to a violation of the sum rule. The strenghts of the trapped states are not perfectly controlled by this rule. However, Fig. 3 convinces one that this violation remains weak in the whole region of λ considered.

The width of the doorway state which acquires with growing λ the main part of the dipole strength lost by the original collective excitation is smaller than those of the other three broad states. Finally, it will be trapped at very large λ (Fig. 2(c)). This behavior of the two states is qualitatively quite similar to that in the two-level approximation (compare Fig. 1). However, an appreciable part of the dipole strength is moved to the other low-lying doorway components which will not be trapped.

Thus, the numerical results confirm the qualitative picture of the interplay of the two kinds of collectivity by which the giant resonance excitations are formed. The coherent internal dipole-dipole residual interaction together with the external interaction via common decay channels creates a concentration of both, the dipole strength and the escape width, on a few collective doorway states.

6 Summary

Summarizing, we state the following. On the basis of a phenomenological schematic model we have investigated the formation of overlapping doorway components of a giant multipole resonance. The interplay and competition of the two kinds of collective behavior induced by the internal and external coupling, respectively, give rise to a nontrivial interference between these components. Two very

different energy scales are formed due to the internal dipole-dipole interaction: In the limit of zero coupling to the continuum all levels with the exception of the collective one are confined to the energy interval Δ_e while the latter is displaced far away by the distance $\mathbf{D}^2 \gg \Delta_e$. With increasing external interaction via k open common decay channels, the width collectivization takes place if TrW exceeds the interval Δ_e . This happens when the main overlapping parameter $\lambda = \langle \gamma \rangle / \mathbf{D}^2$ is still small. As a result, $k + 1$ states get escape widths being comparable to one another while $N - k - 1$ states become trapped. The $k + 1$ states absorbing the total width TrW are the collective doorway states.

When TrW approaches the value \mathbf{D}^2 , with further increasing λ a second stage begins: the widths are redistributed once more being accompanied this time by a strong redistribution of the dipole strength and an energy shift of mainly two doorway states. The width of one of these doorway states starts to decrease and it becomes finally trapped in the limit of very large λ . As a result, a narrow state with a large multipole moment is created due to the external interaction.

The internal damping of the collective motion due to the coupling to complicated compound states has been omitted at this stage. It will be taken into account in part II of this paper where we will also study the influence of the interferences discussed in this paper onto the cross section in order to allow a qualitative comparison with experimental data. It will be shown there that the radiation of the nearly trapped doorway state gives, under certain conditions, the only visible contribution to the γ -ray emission from the giant resonance.

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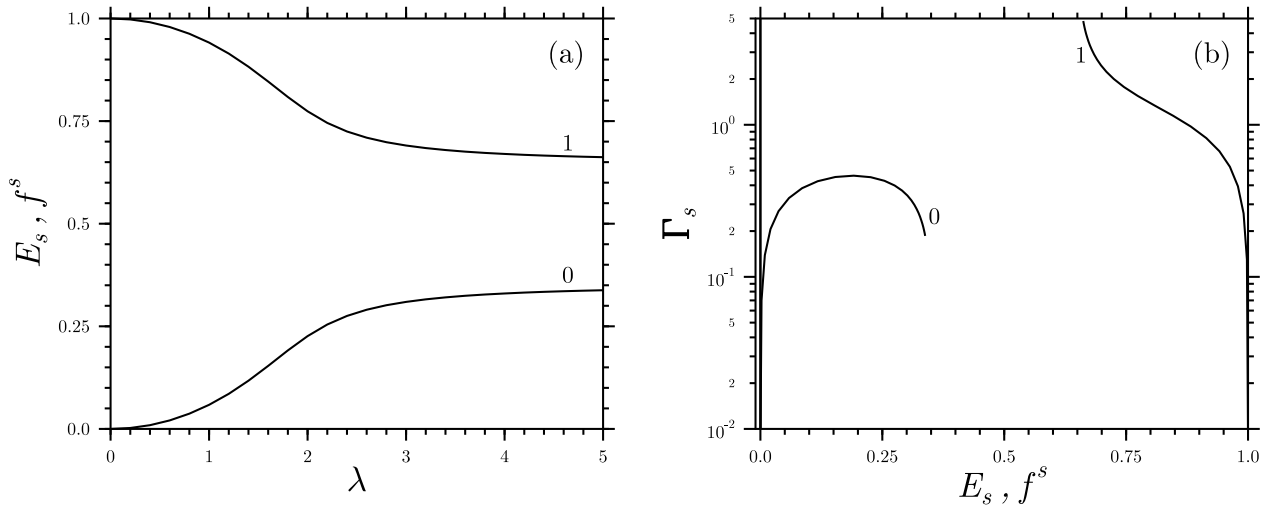


Figure 1: The λ -dependence of resonance energies and dipole strengths (a) and the logarithm of the widths versus energies or strengths (b) with λ varying from 0 to 5 in the two-level approximation.

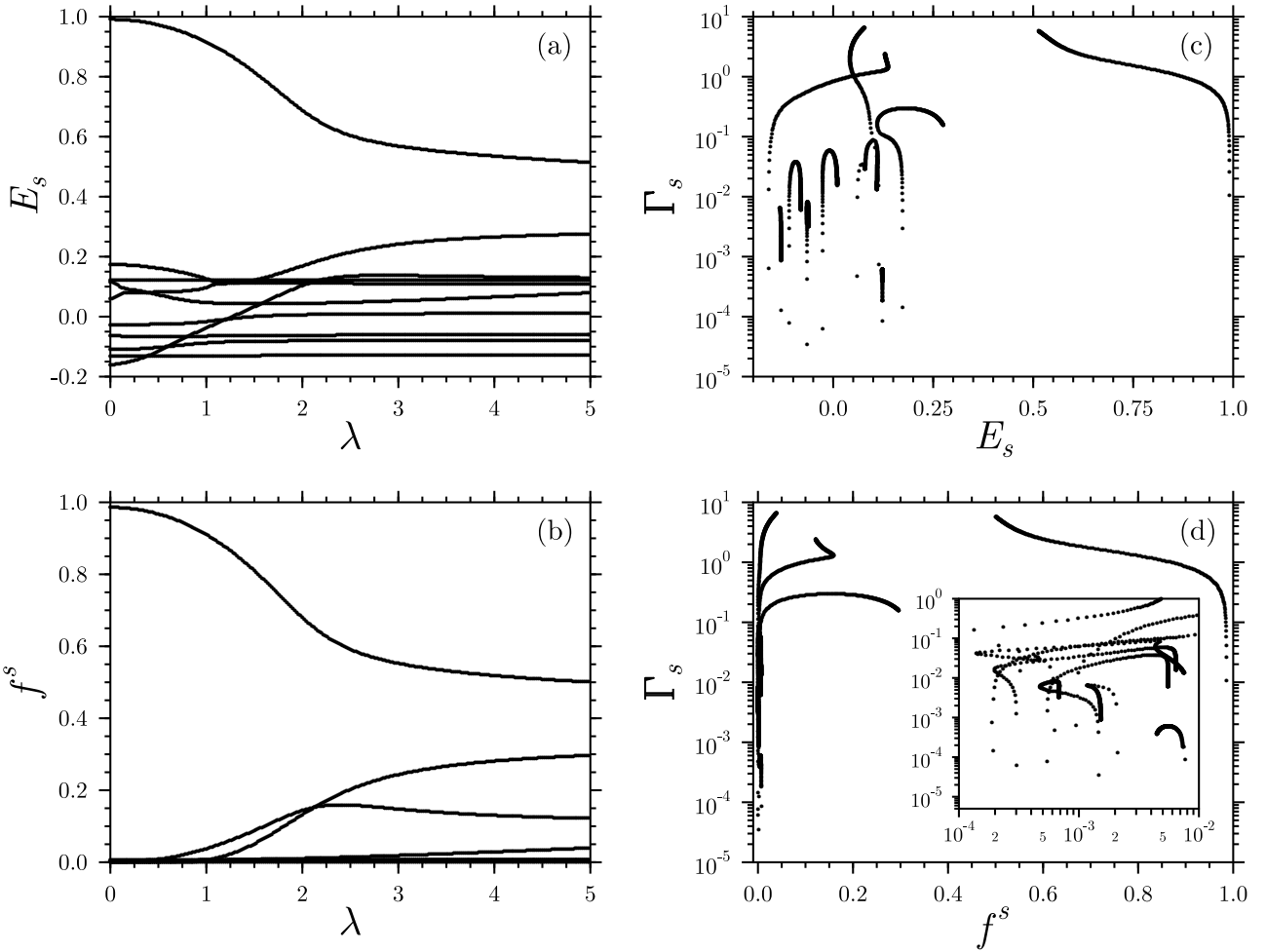


Figure 2: The λ -dependence of resonance energies (a) and dipole strengths (b). The logarithm of the widths versus energies (c) and dipole strengths (d) with λ varying from 0 to 5 in steps of 0.02. (Inset of (d) shows the magnified region of small f^s in the double log-scale). For parameters see text.

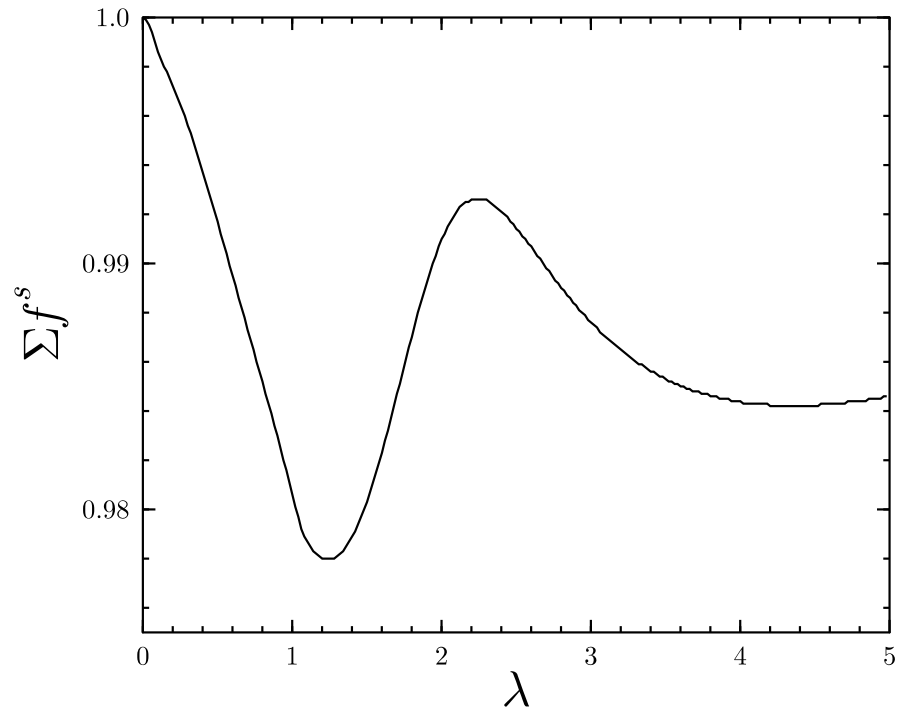


Figure 3: The sum of dipole strengths eqs. (4.29, 4.31) as a function of λ .